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Distributed Passive Fault Tolerant Formation Tracking for Uncertain Second Order Multi-Agent Systems

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Abstract: This paper deals with the problem of distributed passive fault tolerant formation tracking control for cooperative second order multi-agent systems (MASs) subject to disturbances and sensor/actuator faults. The proposed scheme is based on decentralized observers used to robustly estimate the actuator and sensor faults in spite of disturbances. These estimates are then injected into a dynamic control law in order to mitigate their effects on the control objective. Using the $\mathcal{H}_\infty$ method, graph theory properties and the projection lemma, sufficient conditions in the form of a set of linear matrix inequalities (LMIs) are derived to guarantee the stabilization of the tracking errors while reducing the effects of sensor and actuator faults and disturbances. A numerical simulation illustrates the effectiveness of the proposed passive fault-tolerant control scheme.

Keywords: Passive fault-tolerant control, Formation tracking, Multi-agent systems, Actuator faults, Sensor faults.

1. INTRODUCTION

In the past few decades, distributed coordination, control and fault estimation in multi agent systems (MASs) have attracted significant attention as their applications are broad, e.g. satellite formation flying and constellations (Schetter et al., 2003), cooperative unmanned aerial vehicles (Beard et al., 2002) and mobile robots (Liu and Jiang, 2014), to mention a few.

On the other hand, in MASs operating in a vulnerable environment where an extensive integration of communication and computational resources has taken place, robustness against unforeseen events like faults and attacks has become unavoidable. Hence, detecting and mitigating the effects of both types of anomalies has become of paramount importance in control of MASs. Many approaches for the design of fault detection and isolation (FDI) algorithms suitable for such systems have been proposed, e.g (Gao et al., 2018; Wu et al., 2019; Taoufik et al., 2021, 2022). The reader is refereed to (Song and He, 2021) for a recent comprehensive survey.

One of the main issues that arise from the problem of cooperative control for autonomous MASs (Wen et al., 2017), is to develop distributed control protocols such that based on local exchanged information, all agents reach a consensus on certain quantities of interest, in spite of the presence of disturbances and faults. Indeed, such systems are equipped with embedded micro-controllers which are aimed at coordinating data acquisition, communication as well as control actuation. This makes them more vulnerable to actuator and sensor faults or attacks from adversaries that could inject deceptive malicious information into the system through sensors or actuators. Many works have been obtained on this field investigating different types of attacks such as false data injection attacks, e.g. (Hu et al., 2018). In the present paper, the distributed formation tracking control for uncertain second order MASs under actuator and sensor faults is investigated. In order to achieve this, the actuator and sensor faults should be tackled effectively in spite of disturbances. In summary, the main contributions of this paper are twofold

- In contrast with (Khan et al., 2020; Dibaji and Ishii, 2014; Teixeira et al., 2014) where only one type of faults is considered, in this work, a more general uncertain model is considered including sensor and actuator faults and uncertainties. Additionally, the proposed design is not only capable of estimating and isolating the faults, but also mitigates their effects on the control inputs.
- A passive fault tolerant control scheme is proposed whereby an estimator whose practical stability is proved in the presence of perturbations, feeds the fault estimates to a dynamic controller. Some sufficient conditions in the form of LMIs are then derived to guarantee the stabilization of the tracking errors in the closed-loop MAS using $\mathcal{H}_\infty$ method, graph theory properties and the projection lemma, while

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reducing the effects of sensor and actuator faults and disturbances.

The rest of the manuscript is organized as follows. In Section II, the problem formulation is given, namely the modelling of the individual agents, the considered communication topology and the control problem along with some preliminaries. Section III presents the main results of the paper. Section IV illustrates the results through a simulation example.

Notation: Throughout this paper, we denote by Real($[\cdot]$) the real part of $[\cdot]$. For a given square matrix, $\lambda_i([\cdot])$ denotes its ith eigenvalue and $\lambda_{\max}([\cdot])$, $\lambda_{\min}([\cdot])$ its maximum and minimum eigenvalue respectively. For a real matrix $P \in \mathbb{R}^{n\times n}$, $P > 0$ means that P is symmetric and positive-definite. $\mathbf{I}$ is an identity matrix of appropriate dimension. 0 refers to either a null scalar or a matrix with appropriate dimension with all zero entries. The $\mathcal{H}_\infty$ norm of the transfer function $T_{xy}(s)$ linking y to x is defined as $||T_{xy}||_\infty = \sup_{\omega\in\mathbb{R}}\bar{\sigma}(T_{xy}(j\omega))$, where $\bar{\sigma}$ is the maximum singular value of $T_{xy}(s)$. For any square matrix A, $\mathbf{Her}(A) = A+A^T$. The time argument is omitted where it is not needed for clarity.

2. PROBLEM FORMULATION

In this section, the agent's model is laid out as well as some graph theory properties, assumptions and a lemma that are necessary in this work.

Dynamics model: Let us define a MAS composed of a set of N agents labelled $i \in \{1,...,N\}$ and governed by the following double integrator dynamics

$$\begin{cases} \dot{x}_i(t) = v_i(t) + \Delta x_i(t) \\ \dot{v}_i(t) = u_i(t) + f_{u,i}(t) - \tau v_i(t) + \Delta v_i(t) \\ y_i(t) = x_i(t) + f_{s,i}(t) \end{cases} \quad (1)$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$ are the position and the velocity of agent i respectively. $\tau > 0$ is the damping coefficient. $u_i \in \mathbb{R}$ is the control input and $y_i \in \mathbb{R}$ is the measured output signal. Δx_i and Δv_i are some $L2-$norm bounded unknown disturbances/uncertainties. $f_{s,i} \in \mathbb{R}$ and $f_{u,i} \in \mathbb{R}$ represent the additive sensor fault and actuator fault, respectively and might represent attack signals (An and Yang, 2018; Jin et al., 2017). These signals are unknown and can be state dependent. In this work, it is considered that the 2nd derivative of the aggregated faults $f_i(t) = [f_{u,i}, f_{s,i}]^T \in \mathbb{R}^2$ for each agent is $L2-$norm bounded by some unknown variables. In state space form, (1) can be alternatively written as

$$\begin{cases} \dot{\xi}_i(t) = A\xi_i(t) + B_u(u_i(t) + f_{u,i}(t)) + d_i(t) \\ \underline{y}_i(t) = \xi_i(t) + \underline{D}_f f_{s,i}(t) + \bar{D}_f \dot{f}_{s,i}(t) + \underline{D}_d d_i(t) \end{cases} \quad (2)$$

where $\xi_i(t) = [x_i(t)\ \ v_i(t)]^T \in \mathbb{R}^2$, $d_i(t) = [\Delta x_i(t)\ \ \Delta v_i(t)]^T \in \mathbb{R}^2$, $\underline{y}_i^T(t) = [y_i(t)\ \ \dot{y}_i(t)] \in \mathbb{R}^2$, $A = \begin{bmatrix} 0 & 1 \\ 0 & -\tau \end{bmatrix}$, $B_u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\underline{D}_f = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\bar{D}_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\underline{D}_d = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$.

System (2) can then be written as

$$\begin{cases} \dot{\bar{\xi}}_i(t) = \underline{A}\bar{\xi}_i(t) + \bar{B}_u u_i(t) + \bar{B}_\omega \omega_i(t) \\ \bar{y}_i(t) = \bar{C}\bar{\xi}_i(t) + \bar{D}_\omega \omega_i(t) \end{cases} \quad (3)$$

using the following augmented state and output

$$\begin{cases} \bar{\xi}_i(t) = \left[x_i^T(t),\ \zeta_{1,i}^T(t),\ \zeta_{2,i}^T(t)\right]^T \in \mathbb{R}^{\bar{n}} \\ \bar{y}_i(t) = [y_i(t),\quad , \dot{y}_i(t),\quad (\ddot{y}_i(t) - u_i)]^T \in \mathbb{R}^{n_y} \\ \omega_i(t) = [d_i^T(t)\ \ \dot{\Delta} x_i(t)\ \ (f_i^{(2)}(t))^T]^T \in \mathbb{R}^5 \end{cases}$$

with $\zeta_{m,i}(t) = f_i^{(2-m)}(t)$, $\forall m = 1, 2$, where $f_i^{(2)}(t)$ is the second time derivative of $f_i(t)$ and $\bar{n} = 6$, $n_y = 3$. The new state space matrices are as follows

$$\bar{B}_u = \left[B_u^T\ \ 0\ \ 0\right]^T \in \mathbb{R}^{\bar{n}\times 1}$$
$$\bar{B}_g = [0\ \ I_2\ \ 0]^T \in \mathbb{R}^{\bar{n}\times 2}$$
$$\underline{B}_\omega = \begin{bmatrix} I_2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T \in \mathbb{R}^{\bar{n}\times 3}$$
$$\bar{B}_\omega = \left[\underline{B}_\omega\ \ \bar{B}_g\right] \in \mathbb{R}^{\bar{n}\times 5}$$
$$\underline{A} = \begin{bmatrix} A & 0 & [B_u\ \ 0] \\ 0 & 0 & 0 \\ 0 & I_2 & 0 \end{bmatrix} \in \mathbb{R}^{\bar{n}\times\bar{n}}$$
$$\bar{C} = \begin{bmatrix} I_2 & [0\ \ \bar{D}_f] & [0\ \ \underline{D}_f] \\ [0\ \ -\tau] & 0 & [1\ \ 0] \end{bmatrix} \in \mathbb{R}^{n_y\times\bar{n}}$$
$$\bar{D}_\omega = \begin{bmatrix} \underline{D}_d & 0 & 0 \\ [0\ \ 1] & 1 & [0\ \ 1] \end{bmatrix} \in \mathbb{R}^{n_y\times 5}$$

Given system (3), it can be verified that, $\forall s \in \mathcal{C}$, rank $\begin{bmatrix} sI_{\bar{n}} - \bar{A} \\ \bar{C} \end{bmatrix} = \bar{n}$. It is considered that observability and controllability of (2) are preserved when a fault occurs.

Remark 1. It is worth mentioning that $\underline{y}_i(t)$ and thus $\bar{y}_i(t)$, can easily be obtained using a standard robust sliding mode differentiator (Levant et al., 2017).

In the framework of the formation tracking control problem, our main objective is, using local available information and in the presence of faults and perturbations, to drive agents to form a desired geometric shape while tracking a reference trajectory given by

$$\begin{cases} \dot{\xi}_r(t) = A\xi_r(t) + B_u u_r(t) \\ y_r(t) = \xi_r(t) \end{cases} \quad (4)$$

Let us denote by l the size of each agent. The desired formation pattern can be defined using the desired relative inter-agent distance between agent i and the leader as $s_{0_i} = \varrho_i + l$, where ϱ_i is a defined constant. As a result, one can denote $s_{0_{ij}} = \varrho_i - \varrho_j$. Hence, the control objective is to stabilize the tracking disagreement errors to zero, i.e.,

$$\lim_{t\to\infty} ||z_i(t)|| \leqslant \rho, \quad \forall i \in \{1,...,N\} \quad (5)$$

for a small $\rho \in \mathbb{R}^+$, with

$$z_i = \begin{bmatrix} z_{1i} \\ z_{2i} \end{bmatrix} = \xi_i - \xi_r - \begin{bmatrix} s_{0_i} \\ 0 \end{bmatrix} \quad (6)$$

Assumption 1. It is assumed that at least one agent has direct access to the leader's state. Additionally, it assumed that the followers can obtain an exact estimate of the leader's state using a fixed-time observer (Taoufik et al., 2021, 2020).

Graph theory: In this work, the interaction between agents is modelled using graph theory. Let us denote by $\mathcal{Q} = (\mathcal{N}, \mathcal{F})$ the representation of the communication topology composed of N interacting agents described with the double integrator dynamics (1), $\mathcal{N} = \{1,...,N\}$ is the node set consisting of N nodes representing each agent.

$\mathcal{F} \subseteq \mathcal{N} \times \mathcal{N}$ is the fixed edge set representing the communication links between the agents. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined by $a_{ij} > 0$ when the ith agent can receive information from the jth agent and $a_{ij} = 0$ otherwise, and $a_{ii} = 0$. Let $\mathcal{D}$ be the in-degree diagonal matrix, with $d_{in_i} = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix $\mathcal{L}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathcal{R}^{N\times N}$. In this work, we make the assumption that $\mathcal{Q}$ is connected and has a spanning tree, $\mathcal{N}_i \subset \{1, ..., N\}\backslash\{i\}$ is the non-empty subset of agents that agent i can sense and interact with. Let us denote by $\mathcal{L}_r = \text{diag}(\ell_1^r, \ell_2^r, ...\ell_N^r)$ the diagonal matrix defining the interconnections between the leader and the followers where $\ell_i^r > 0$ when agent i can access the leader's information. The leader agent does not receive any information, whereas all the followers know the input of the leader. In this work, the following assumption is considered.

Assumption 2. The communication topology described by graph $\mathcal{Q}$ is either

- undirected, or
- directed with a spanning tree, where $\bar{\mathcal{L}} = \mathcal{L} + \mathcal{L}_r$ is diagonalisable (Li et al., 2018).

In the next section, a passive fault tolerant leader-follower tracking control protocol is proposed to achieve objective (5), in spite of the presence of faults, perturbations and damping effect, using an observer-based technique. The following lemma will be used in the next section.

Lemma 1. (Gahinet and Apkarian, 1994) Given a symmetric matrix Z and two matrices $\mathcal{U}$ and $\mathcal{V}$ of appropriate dimensions. There exists a matrix Υ satisfying

$$\mathcal{Z} + \mathcal{U}^T\Upsilon\mathcal{V} + \mathcal{V}^T\Upsilon^T\mathcal{U} < 0,$$

if and only if the projection inequalities $N_{\mathcal{U}}^T\mathcal{Z}N_{\mathcal{U}} < 0$, $N_{\mathcal{V}}\mathcal{Z}N_{\mathcal{V}} < 0$, with respect to Υ are satisfied, where $N_{\mathcal{U}}$ and $N_{\mathcal{V}}$ are arbitrary matrices forming the right orthogonal complements of $\mathcal{U}$ and $\mathcal{V}$, respectively.

3. PASSIVE FAULT TOLERANT CONTROL DESIGN

Let us consider the following observer-based dynamic control law using the available information

$$\dot{\hat{\bar{\xi}}}_i = \underline{A}\hat{\bar{\xi}}_i + \bar{B}_u u_i + L(\bar{y}_i - \bar{C}\hat{\bar{\xi}}_i) \tag{7a}$$

$$\begin{aligned}\dot{s}_i &= (A + B_uK)s_i \\ &+ W\Big[\sum_{j\in\mathcal{N}_i} a_{ij}(s_i - s_j) + \ell_i^r(s_i - s_r) - \delta_i\Big]\end{aligned} \tag{7b}$$

$$u_i = Ks_i - \Psi\hat{\bar{\xi}}_i + [0\ 1]\,(\dot{y}_r - A\xi_r) \tag{7c}$$

where $\hat{\bar{\xi}}_i \in \mathcal{R}^{\bar{n}}$ is the augmented state estimate for agent i and $L \in \mathcal{R}^{\bar{n}\times n_y}$ is the observer gain matrix. $s_i \in \mathcal{R}^2$ is the control protocol state. $K \in \mathcal{R}^{1\times 2}$ and $W \in \mathcal{R}^{2\times 2}$ are controller gains to be designed. $\dot{s}_r = (A + B_uK)s_r$ and $\Psi = [0_{1\times 4}\ 1\ 0]$. δ_i represents the relative measurements by agent i with respect to its neighbours, and is expressed as

$$\begin{aligned}\delta_i = &\Big[\sum_{j\in\mathcal{N}_i}(a_{ij}(\xi_i - \xi_j - \Sigma_{ij}) + \ell_i^r(\xi_i - \xi_r - \Sigma_i))\Big] \\ &+\Big[\sum_{j\in\mathcal{N}_i}(a_{ij}(e_{f_{s,i}} - e_{f_{s,j}}) + \ell_i^r e_{f_{s,i}})\Big] \\ &+\underline{D}_d\Big[\sum_{j\in\mathcal{N}_i}(a_{ij}(d_i - d_j) + \ell_i^r d_i)\Big]\end{aligned}$$

with $\Sigma_{ij} = \begin{bmatrix} s_{0_{ij}} \\ 0 \end{bmatrix}$, $\Sigma_i = \begin{bmatrix} s_{0_i} \\ 0 \end{bmatrix}$. $\bar{\Gamma} = \begin{bmatrix} 0_{1\times 3} & 0 & 0 & 1 \\ 0_{1\times 3} & 1 & 0 & 0 \end{bmatrix}$. The sensor fault estimation error is given as $e_{f_{s,i}} = \begin{bmatrix} f_{s,i}(t) \\ \dot{f}_{s,i}(t) \end{bmatrix} - \bar{\Gamma}\hat{\bar{\xi}}_i(t)$.

Remark 2. It is worth noting that in the proposed passive fault tolerant control structure, the estimators are decoupled from the dynamic controller. Indeed, both modules can be successively designed as highlighted hereafter.

By selecting the variable $\varepsilon_i = s_i - s_r$, an additional control objective is set out as

$$\lim_{t\to\infty} ||\varepsilon_i|| = 0,\ \forall i \in \{1, ..., N\} \tag{8}$$

Using the fact that $A\Sigma_i = 0$, the errors can be expressed as

$$\begin{aligned}\dot{z}_i &= Az_i + B_uK\varepsilon_i + B_u e_{f_{u,i}} + d_i \\ \dot{\varepsilon}_i &= (A + B_uK)\varepsilon_i \\ &+W\Bigg[\sum_{j\in\mathcal{N}_i}\big(a_{ij}(\varepsilon_i - \varepsilon_j) + \ell_i^r\varepsilon_i\big) - \sum_{j\in\mathcal{N}_i} a_{ij}e_{f_{s,j}} \\ &- \sum_{j\in\mathcal{N}_i}\big(a_{ij}(z_i - z_j) + \ell_i^r z_i\big) - (d_{in_i} + \ell_i^r)e_{f_{s,i}} \\ &-\underline{D}_d(d_{in_i} + \ell_i^r)d_i - \underline{D}_d\sum_{j\in\mathcal{N}_i} a_{ij}d_j\Bigg]\end{aligned} \tag{9}$$

with $B_u f_{u,i}(t) - B_u\Psi\hat{\bar{\xi}}_i = B_u e_{f_{u,i}}$, where $e_{f_{u,i}}$ represents the actuator estimation error. By letting $\varphi_i^T = [z_i^T, \varepsilon_i^T]$, $\varphi_g^T = [\varphi_1^T, ..., \varphi_N^T]$, $\varepsilon_g = [\varepsilon_1, ..., \varepsilon_N]$, the global closed-loop system is given as

$$\dot{\Xi} = \big(\mathbf{I}_N \otimes (\underline{A} - L\bar{C})\big)\Xi - \big(\mathbf{I}_N \otimes (\bar{B}_\omega + L\bar{D}_\omega)\big)\omega \tag{10a}$$

$$\begin{aligned}\dot{\varphi}_g &= \big(\mathbf{I}_N \otimes \bar{A} + \bar{\mathcal{L}} \otimes \bar{W}\big)\varphi_g + (\mathbf{I}_N \otimes \bar{B}_{e_u})e_u \\ &+ (\bar{\mathcal{L}} \otimes \bar{B}_{e_s})e_s + (\mathbf{I}_N \otimes \bar{B}_{d,1} + \bar{\mathcal{L}} \otimes \bar{B}_{d,2})d\end{aligned} \tag{10b}$$

where $e_i = \hat{\bar{\xi}}_i - \bar{\xi}_i$, $\Xi^T = [e_1^T, ..., e_N^T]$, $\omega^T = [\omega_1^T, ..., \omega_N^T]$, $e_u^T = [e_{f_{u,1}}^T, ..., e_{f_{u,N}}^T]$, $e_s^T = [e_{f_{s,1}}^T, ..., e_{f_{s,N}}^T]$, $d^T = [d_1^T, ..., d_N^T]$, $\bar{B}_{e_u} = \begin{bmatrix} B_u \\ 0 \end{bmatrix}$, $\bar{B}_{e_s} = \begin{bmatrix} 0 \\ -W \end{bmatrix}$, $\bar{B}_{d,1} = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}$, $\bar{B}_{d,2} = \begin{bmatrix} 0 \\ -W\underline{D}_d \end{bmatrix}$, $\bar{A} = \begin{bmatrix} A & B_uK \\ 0 & A + B_uK \end{bmatrix}$ and $\bar{W} = \begin{bmatrix} 0 & 0 \\ -W & W \end{bmatrix}$.

Hence, the passive fault-tolerant formation tracking objective is achieved by stabilizing both (10a) and (10b). The next theorem provides conditions w.r.t. L such that the practical stability of system (10a) is guaranteed.

Theorem 1. The error dynamics (10a) are practically stable, if the following is satisfied

$$\begin{aligned}&(\beta\mathbf{I} + \big(\mathbf{I}_N \otimes \underline{A}^T\big))\bar{Q} + \bar{Q}(\beta\mathbf{I} + \big(\mathbf{I}_N \otimes \underline{A}\big)) \\ &= \big(\mathbf{I}_N \otimes \bar{C}^T\bar{C}\big), \qquad \bar{\sigma}(L\bar{D}_\omega) < \mu\end{aligned} \tag{11}$$

for $\beta > 0$ satisfying $\text{Real}[\lambda_p(\bar{A})] > -\beta$, $\forall p \in \{1, 2..., 6\}$, $Q > 0$, where $\bar{Q} = \mathbf{I}_N \otimes Q$, $L = Q^{-1}\bar{C}^T$, and μ is minimised.

Proof 1. Consider the Lyapunov function $V_{\bar{\xi}} = \Xi^T\bar{Q}\Xi$, then

$$\begin{aligned}\dot{V_{\bar{\xi}}} &= \Xi^T\big[\big(\mathbf{I}_N\otimes(\underline{A}-L\bar{C})\big)^T\bar{Q}+\bar{Q}\big(\mathbf{I}_N\otimes(\underline{A}-L\bar{C})\big)\big]\Xi\\ &\quad -2\Xi^T\bar{Q}\big(\mathbf{I}_N\otimes(\bar{B}_\omega+L\bar{D}_\omega)\big)\omega\\ &\leqslant -2\beta V_{\bar{\xi}}\\ &\quad +2\|\Xi^T\sqrt{\bar{Q}}\|\cdot\|\sqrt{\bar{Q}}\big(\mathbf{I}_N\otimes(\bar{B}_\omega+L\bar{D}_\omega)\big)\omega\|\end{aligned}$$

Hence $d\sqrt{V_{\bar{\xi}}}/dt \leqslant -\beta\sqrt{V_{\bar{\xi}}}+\|\sqrt{\bar{Q}}\big(\mathbf{I}_N\otimes(\bar{B}_\omega+L\bar{D}_\omega)\big)\omega\|$. Therefore, if

$$\|\Xi\| \leqslant \beta^{-1}\frac{\lambda_{\min}(\sqrt{\bar{Q}})}{\lambda_{\max}(\sqrt{\bar{Q}})}\|\big(\mathbf{I}_N\otimes(\bar{B}_\omega+L\bar{D}_\omega)\big)\|\|\omega\|$$

then $V_{\bar{\xi}}$ is decreasing. Consequently, (10a) is practically stable and the gain β is used to tune the upper bound of the error vector.

Next, an LMI formulation to stabilize the global system (10b), i.e., to achieve objective (5) and (8) is proposed, where an adequate rejection performance of the unknown input fault estimation error and disturbances is achieved, by guaranteeing:

$$\|T_{\varphi_g e_u}\|_\infty < \sigma_u, \|T_{\varphi_g e_s}\|_\infty < \sigma_s, \|T_{\varphi_g d}\|_\infty < \sigma_d \quad (12)$$

where σ_u, σ_s and σ_d are variables to be minimised. In order to achieve the performance constraint (12), let us first decouple system (10b) to N subsystems (i.e., remove the term $\bar{\mathcal{L}}$). For this, consider a unitary matrix $J\in\mathbb{R}^{N\times N}$ such that $\bar{\mathcal{L}}$ such that $\Gamma = J^{-1}\bar{\mathcal{L}}J = \text{diag}[\lambda_1,...,\lambda_N]$, where λ_i is the ith eigenvalue of $\bar{\mathcal{L}}$. Hence, given the associative property of the Kronecker product, we use the change of coordinates

$$\begin{cases}\tilde{\varphi}_g &= (J^{-1}\otimes\mathbf{I})\varphi_g &= [\tilde{\varphi}_1^T,...,\tilde{\varphi}_N^T]^T\\ \tilde{e}_s &= (J^{-1}\otimes\mathbf{I})e_s &= [\tilde{e}_{s,1}^T,...,\tilde{e}_{s,N}^T]^T\\ \tilde{e}_u &= (J^{-1}\otimes\mathbf{I})e_u &= [\tilde{e}_{u,1}^T,...,\tilde{e}_{u,N}^T]^T,\\ \tilde{d} &= (J^{-1}\otimes\mathbf{I})d &= [\tilde{d}_1^T,...,\tilde{d}_N^T]^T\end{cases} \quad (13)$$

Given that $J\otimes\mathbf{I}$ is a unitary matrix (Lin et al., 2008), one has $\forall i$, $\|T_{\varphi_g e_u}\|_\infty = \max_i\|T_{\tilde{\varphi}_i\tilde{e}_{u,i}}\|_\infty$, $\|T_{\varphi_g e_s}\|_\infty = \max_i\|T_{\tilde{\varphi}_i\tilde{e}_{s,i}}\|_\infty$ and $\|T_{\varphi_g d}\|_\infty = \max_i\|T_{\tilde{\varphi}_i\tilde{d}_i}\|_\infty$, where we define the $\mathcal{H}_\infty$ performance norms as

$$\|T_{\tilde{\varphi}_i\tilde{e}_{u,i}}\|_\infty = \|(s\mathbf{I}-F_i)^{-1}\bar{B}_{e_u}\|_\infty < \sigma_u \quad (14a)$$

$$\|T_{\tilde{\varphi}_i\tilde{e}_{s,i}}\|_\infty = \|(s\mathbf{I}-F_i)^{-1}\bar{B}_1\|_\infty < \sigma_s \quad (14b)$$

$$\|T_{\tilde{\varphi}_i\tilde{d}_i}\|_\infty = \|(s\mathbf{I}-F_i)^{-1}\bar{B}_2\|_\infty < \sigma_d \quad (14c)$$

where $F_i = \bar{A}+\lambda_i\bar{W}$, $\bar{B}_1 = \lambda_i\bar{B}_{e_s}$ and $\bar{B}_2 = \bar{B}_{d,1}+\lambda_i\bar{B}_{d,2}$.

Theorem 2. Given a communication topology satisfying Assumptions 1-2. Let σ_u, σ_s and σ_d be strictly positive scalars and associated strictly positive multi-objective weights π_u, π_s and π_d, system (10b) is stable and the perturbation and fault error rejection performance indexes (12) are guaranteed if $\forall i\in\{1,2,...,N\}$ there exist symmetric matrices $P_i>0$ and matrices $Y = \text{Blkdiag}(Y_1,Y_1)$, Q_1, Q_2 such that the following optimisation problem is solved for each matrix $\Lambda\in\{\Lambda_u,\Lambda_s,\Lambda_d\}$ and corresponding scalar $\sigma_\Delta\in\{\sigma_u,\sigma_s,\sigma_d\}$

$$\min_{P_i,Q_1,Q_2,Y}\pi_u\sigma_u+\pi_s\sigma_s+\pi_d\sigma_d$$

subject to

$$\begin{bmatrix}-Y-Y^T & * & * & * & *\\ \Theta_i+P_i & -P_i & * & * & *\\ \Lambda & 0 & -\mathbf{I} & * & *\\ Y & 0 & 0 & -P_i & *\\ 0 & \mathbf{I} & 0 & 0 & -\sigma_\Delta^2\mathbf{I}\end{bmatrix} < 0 \quad (15)$$

with

$$\begin{aligned}\Theta_i &= \begin{bmatrix}A^TY_1 & -\lambda_iQ_2\\ Q_1B_u^T & A^TY_1+\lambda_iQ_2+Q_1B_u^T\end{bmatrix}\\ \Lambda_u &= \begin{bmatrix}B_u^TY_1 & 0\end{bmatrix}\\ \Lambda_s &= \begin{bmatrix}0 & -\lambda_iQ_2\end{bmatrix}\\ \Lambda_d &= \begin{bmatrix}Y_1 & -\lambda_i\underline{D}_d^TQ_2\end{bmatrix}\end{aligned} \quad (16)$$

Then, the control gain matrices K, W are computed as $W = (Y_1^{-1}Q_2)^T$ and $K = (\tilde{Y}_1^{-1}Q_1)^T$ with $B_u^TY_1 = \tilde{Y}_1B_u^T$.

Proof 2. Using (13), one has

$$\begin{aligned}\dot{\tilde{\varphi}}_g &= \big(\mathbf{I}_N\otimes\bar{A}+\Gamma\otimes\bar{W}\big)\tilde{\varphi}+(\mathbf{I}_N\otimes\bar{B}_{e_u})\tilde{e}_u\\ &\quad+(\Gamma\otimes\bar{B}_{e_s})\tilde{e}_s+(\mathbf{I}_N\otimes\bar{B}_{d,1}+\Gamma\otimes\bar{B}_{d,2})\tilde{d}\end{aligned} \quad (17)$$

Now, consider the Lyapunov function, $V_{\tilde{\varphi}} = \tilde{\varphi}_g{}^TX\tilde{\varphi}_g$. First consider the case where $e_s = e_u = 0$ and let $X = \text{Blkdiag}(X_1,...,X_N)$, $X_i>0$, $\forall i = 1,...,N$ then

$$\begin{aligned}\dot{V}_{\tilde{\varphi}} &= \mathbf{Her}\Big(\tilde{\varphi}_g^T\big(X\otimes\bar{A}+X\Gamma\otimes\bar{W}\big)\tilde{\varphi}_g+\tilde{\varphi}_g^T(X\otimes\bar{B}_{d,1}\\ &\quad+X\Gamma\otimes\bar{B}_{d,2})\tilde{d}\Big)\end{aligned} \quad (18)$$

It follows from $V_{\tilde{\varphi}} = \sum_i^N V_{\tilde{\varphi}_i}$ that (18) is equivalent to

$$\begin{aligned}\dot{V}_{\tilde{\varphi}_i} &= \tilde{\varphi}_i^TF_i^TX_i\tilde{\varphi}_i+\tilde{d}_i^T\bar{B}_2^TX_i\tilde{\varphi}_i+\tilde{\varphi}_i^TX_iF_i\tilde{\varphi}_i+\tilde{\varphi}_i^TX_i\bar{B}_2\tilde{d}_i,\\ &\quad\forall i\in\{1,...,N\},\end{aligned} \quad (19)$$

Furthermore, to satisfy the defined performance index (14c), one has to minimize the function

$$\mathcal{J}_i = \int_0^\infty(\tilde{\varphi}_i^T\tilde{\varphi}_i-\sigma_d^2\tilde{d}_i^T\tilde{d}_i)dt < 0 \quad (20)$$

Combining (19) and (20) yields

$$\begin{bmatrix}P_iF_i^T+F_iP_i-P_i+\sigma_d^{-1}\bar{B}_2\bar{B}_2^T & P_i & P_i\\ * & -\sigma_d\mathbf{I} & 0\\ * & * & -P_i\end{bmatrix} < 0 \quad (21)$$

where $P_i = X_i^{-1}$. It is easy to notice that (21) can be equivalently expressed as $N_{\mathcal{U}_i}^T\mathcal{Z}_iN_{\mathcal{U}_i}<0$, where

$$\mathcal{Z}_i = \begin{bmatrix}0 & P_i & 0 & 0 & 0\\ P_i & -P_i & 0 & 0 & 0\\ 0 & 0 & -\sigma_d\mathbf{I} & 0 & 0\\ 0 & 0 & 0 & -P_i & 0\\ 0 & 0 & 0 & 0 & -\sigma_d\mathbf{I}\end{bmatrix}, N_{\mathcal{U}_i} = \begin{bmatrix}F_i^T & \mathbf{I} & \mathbf{I}\\ \mathbf{I} & 0 & 0\\ 0 & \mathbf{I} & 0\\ 0 & 0 & \mathbf{I}\\ \sigma_d^{-1}\bar{B}_2 & 0 & 0\end{bmatrix}$$

Hence, by defining

$$N_{\mathcal{V}}^T = \begin{bmatrix}0 & \mathbf{I} & 0 & 0 & 0\\ 0 & 0 & \mathbf{I} & 0 & 0\\ 0 & 0 & 0 & \mathbf{I} & 0\end{bmatrix}, \mathcal{U}_i = \begin{bmatrix}-\mathbf{I} & F_i^T & \mathbf{I} & \mathbf{I} & 0\\ 0 & \bar{B}_2^T & 0 & 0 & -\sigma_d\mathbf{I}\end{bmatrix}$$

It can be verified that $N_{\mathcal{V}}^T\mathcal{Z}_iN_{\mathcal{V}} = -\text{diag}(P_i,\sigma_d\mathbf{I},P_i)<0$. By invoking Lemma 1, one can deduce that solving $N_{\mathcal{U}_i}^T\mathcal{Z}_iN_{\mathcal{U}_i}<0$ is equivalent to $\mathcal{Z}_i+N_{\mathcal{V}}^T\Upsilon N_{\mathcal{V}}+N_{\mathcal{U}_i}^T\Upsilon N_{\mathcal{U}_i}<0$. Hence, by choosing $\Upsilon = \text{Blkdiag}(Y,\mathbf{I})$, (**??**) becomes

$$\begin{bmatrix}-Y-Y^T & * & * & * & *\\ F_iY+P_i & -P_i & * & * & *\\ Y & 0 & -\mathbf{I} & * & *\\ Y & 0 & 0 & -P_i & *\\ 0 & \bar{B}_2^T & 0 & 0 & -\sigma_d^2\mathbf{I}\end{bmatrix} < 0 \quad (22)$$

By applying the transformation $(F_i,\bar{B}_2,\mathbf{I})\to(F_i^T,\mathbf{I},\bar{B}_2^T)$, and by setting $Y = \text{Blkdiag}(Y_1,Y_1)$, $B_u^TY_1 = \tilde{Y}_1B_u^T$ and

$$\begin{aligned}\Theta_i &= \begin{bmatrix}A^TY_1 & -\lambda_iW^TY_1\\ K^T\tilde{Y}_1B_u^T & A^TY_1+\lambda_iW^YY_1+K^T\tilde{Y}_1B_u^T\end{bmatrix}\\ \Lambda_d &= \begin{bmatrix}Y_1 & -\lambda_i\underline{D}_d^TW^TY_1\end{bmatrix}\end{aligned} \quad (23)$$

then (15) is obtained for $\Lambda = \Lambda_d$, $\sigma_\Delta = \sigma_d$ through the changes of variables $Q_1 = K^T\tilde{Y}_1$ and $Q_2 = W^T Y_1$. Following the same reasoning for indexes (14b) and (14c) one can conclude the proof.

Note that the multi-objective weights π_u, π_s and π_d are design variables used to give more importance to a chosen criteria in the optimisation problem (15) over the others.

4. ILLUSTRATIVE EXAMPLE

In this section, as a scenario to illustrate the feasibility of the proposed observer-based controller, consider a team of five robots governed by the double integrator dynamics (1) with $\tau = 0.1$. The initial conditions are

$$\begin{aligned} x_1(0) &= 9m, & v_1(0) &= 0m/s \\ x_2(0) &= 7.5m, & v_2(0) &= 0m/s \\ x_3(0) &= 5m, & v_3(0) &= 0m/s \\ x_4(0) &= 2.5m, & v_4(0) &= 0m/s \\ x_5(0) &= 0.5m, & v_5(0) &= 0m/s \end{aligned}$$

They interact with each other according to the line-like communication topology illustrated in Fig. 1 and defined by the following matrices

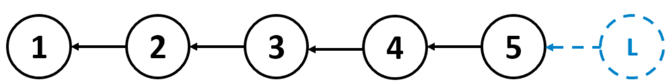

Fig. 1. Communication graph.

$$\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{L}_r = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&1 \end{bmatrix}$$

where node L represents the leader. It is worth noticing that in this example, Assumption 2 is fulfilled and that all the eigenvalues λ_i of $\bar{\mathcal{L}}$, are equal to 1, $\forall i \in \{1, ..., 5\}$. Hence Theorem 2 is reduced to the resolution of three LMIs. Here, it is considered that a distance of $1m$, $\forall i \in \{1, 2, ..., N\}$ is kept between each two consecutive agents. Their size is defined as $l = 2m$, hence guaranteeing a total distance of $3m$ between their centre of gravity, i.e.,

$$s_{0_i} = 3(N + 1 - i)$$

Applying Theorem 2 yields the controller gains

$$K = [-20.6940 \;\; -28.2420]$$

$$W = \begin{bmatrix} -1.2938 & -1.7538 \\ -0.0369 & -3.9924 \end{bmatrix}$$

Here, it is considered that two faults $f_{u,4}$ and $f_{s,2}$ occur in the network for agent 4 and 2 respectively as shown in Figure 2, where $f_{u,4}$ is a persistent actuator fault affecting the input of robot 4 and $f_{s,2}$ is a temporary sensor fault affecting the sensor of agent 2. Moreover, the disturbance signals are given as

$$d_i(t) = \begin{bmatrix} \Delta x_i(t) \\ \Delta v_i(t) \end{bmatrix} = \begin{bmatrix} 0.1i\sin(10t) \\ 0.1i\cos(10t) \end{bmatrix}$$

The reference signals $x_r(t)$ and $v_r(t)$ are represented in Figures 4 and 5 respectively. One can see from Figure 2 that the proposed scheme can robustly estimate the fault signals. Figures 3 and 4 depict the two elements of the disagreement vector $z_i(t) = [z_{1i}(t) \; z_{2i}(t)]^T$, for each robot. It can be noticed that the system achieves formation tracking in spite of both fault signals and disturbances and that the effects of the fault signals are well-mitigated. Figure 5 shows the position and velocity for each agent and the corresponding reference signal.

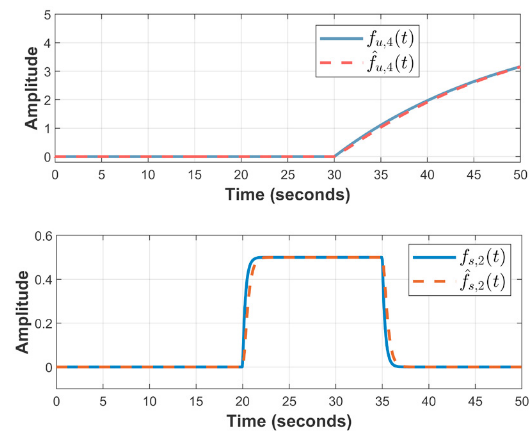

Fig. 2. Fault signals and their estimates.

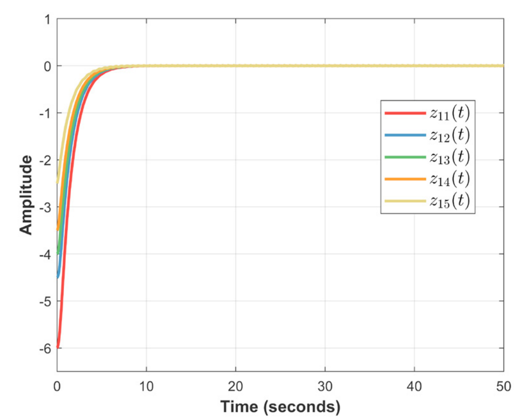

Fig. 3. First elements of the disagreement vector.

5. CONCLUSION

In this paper, the problem of passive fault tolerant formation tracking controller design for cooperative multi-agent systems with double integrator dynamics subject to disturbances and sensor/actuator faults has been investigated. The problem was solved using decentralized observers to robustly estimate the actuator and sensor faults in spite of disturbances. An observer-based controller has been designed to track the reference trajectory while maintaining a desired formation pattern. Using the $\mathcal{H}_\infty$ method, graph theory properties and the projection lemma, some sufficient conditions in the form of LMI have been derived to guarantee the stabilization of the tracking errors while reducing the effects of sensor and actuator faults and disturbances. Finally, the efficacy of the proposed design has been shown through an illustrative numerical example.

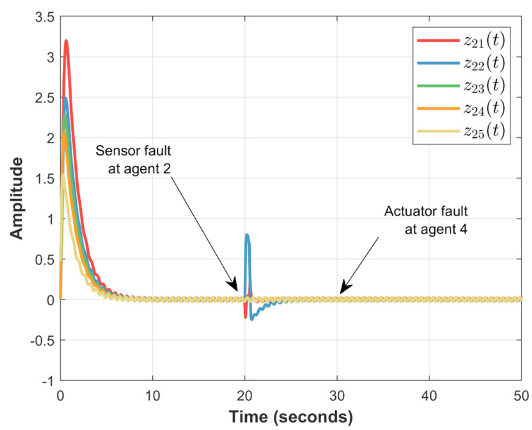

Fig. 4. Second elements of the disagreement vector.

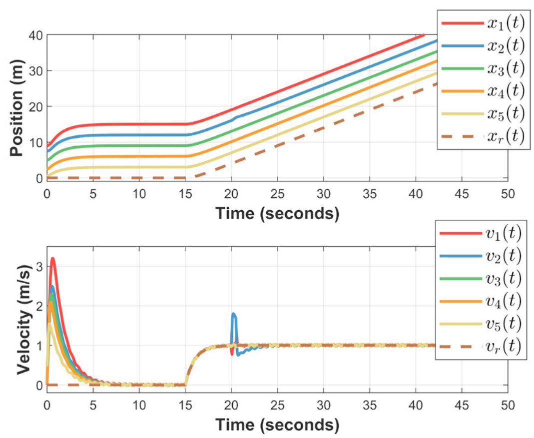

Fig. 5. Position and velocity for each agent with the reference signal.

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